$$\tau_2 = I_2(\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_{c2} \sin q_2 \, \dot{q}_1^2$$
$$+ m_2 l_1 l_{c2} \cos q_2 \, \ddot{q}_1 + m_2 l_{c2}^2 (\ddot{q}_1 + \ddot{q}_2)$$
$$+ m_2 l_{c2} \, g \cos(q + q_2) \qquad (a)$$

$$\tau_1 = \tau_2 + m_1 l_{c1}^2 + m_1 l_{c1} g \cos q_1 + I_1 \ddot{q}_1$$
$$+ m_2 l_1^2 \ddot{q}_1 - m_1 l_1 l_{c2} (\dot{q}_1 + \dot{q}_2)^2 \sin q_2$$
$$+ m_2 l_1 l_{c2} (\ddot{q}_1 + \ddot{q}_2) \cos q_2 \qquad (b)$$

Control of Robot Manipulator

The majority of robot manipulators in practice are PD control. We start with very simple control. Consider an N-DOF rigid robot manipulator.

$$\ddot{q}_t D(q) + C(q, \dot{q})\dot{q} + G(q) = \tau$$

$$\tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix} \qquad q = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix}$$

Let $$q^d(t) = \begin{bmatrix} q_1^d(t) \\ \vdots \\ q_n^d(t) \end{bmatrix}$$

trajectory (Desired Trajectory)

Design the control input τ such that q is bounded and $\lim_{t \to \infty} [q(t) - q^d(t)] = 0$

Let us assume that there is some trajectory error.

$$\tilde{q} = q^d - q$$

Assume that the robot parameters are known (i.e. $D(q), C(q,\dot{q}), G(q)$ are known)

<u>Computed torque control</u>

let

$$\tau = C(q,\dot{q})\dot{q} + G(q) + D(q)V \quad (2)$$

V will be designed later.

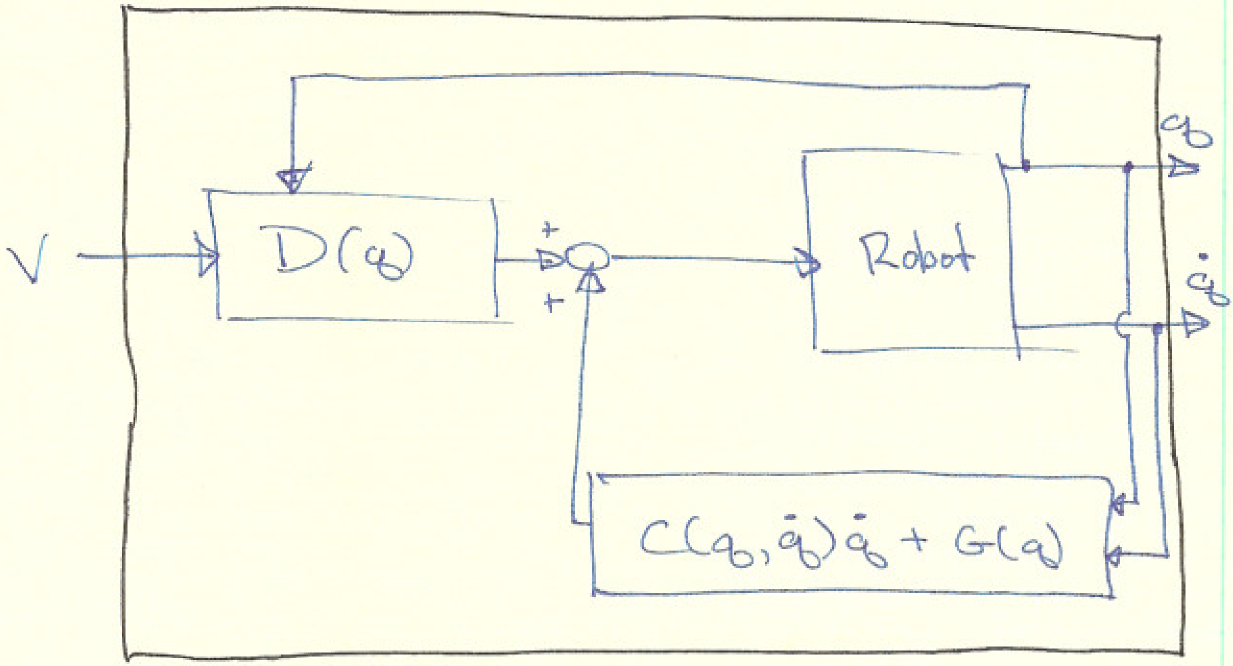
(1) & (2) together

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)$$
$$= C(q,\dot{q})\dot{q} + G(q) + D(q)V$$

$$D(q)\ddot{q} = D(q)V$$

Since $\det(D(q)) \neq 0$

$$\boxed{\ddot{q} = V}$$



$$= \frac{1}{s^2}$$

Note: It is not really this easy.

$$\ddot{q} = V$$

$$V = \ddot{q}_d + K_v \dot{\tilde{q}} + K_p \tilde{q}$$

$$K_p = \begin{bmatrix} k_{p1} & & \\ & k_{p2} & \\ & & \ddots \end{bmatrix} \qquad K_v = \begin{bmatrix} K_{v1} & & \\ & \ddots & \\ & & K_{v2} \end{bmatrix}$$

$K_{pi} > 0 \qquad K_{vi} > 0$

$i = 1 \ldots n$

$i' = 1 \ldots n$

5 ξ 4

$$\ddot{q} = \ddot{q}_D + K_v \dot{\tilde{q}} + K_p \tilde{q}$$

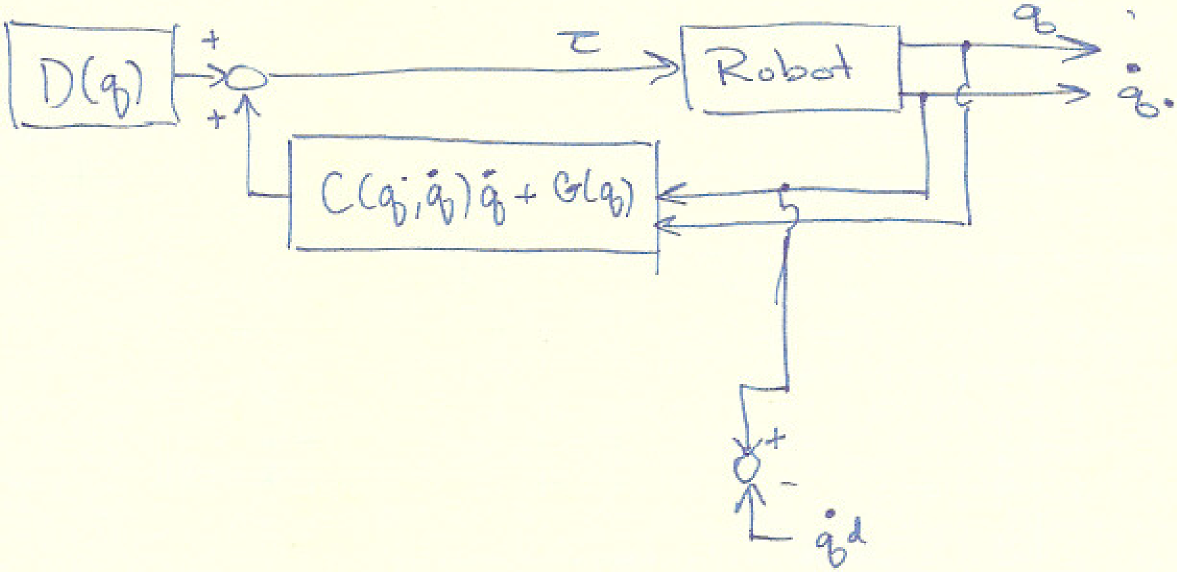
$$\ddot{\tilde{q}} + K_v \dot{\tilde{q}} + K_p \tilde{q} = 0$$

$$\therefore \quad \ddot{\tilde{q}}_i + K_{vi} \dot{\tilde{q}} + K_p \tilde{q} = 0$$

$$i = 1 \ldots n$$

$$\boxed{\begin{array}{l} \tilde{q}_i \text{ is banded} \\ \lim_{t \to \infty} \tilde{q}_i(t) = 0 \end{array}}$$

When
$K_{vi} > 0$
$K_{pi} > 0$



PD & gravity Compensation.

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

this does not consider friction.

this is all we know

$$q_d = \begin{bmatrix} q_1^d \\ \vdots \\ q_n^d \end{bmatrix}$$

desired trajectory
assumed constant.

$$\tilde{q} = q_d - q$$

let $V(q, \dot{q})$ be the lyapunov function

$$V(\tilde{q}, \dot{\tilde{q}}) = \frac{1}{2} \tilde{q}^T k_p \tilde{q} + \frac{1}{2} \dot{\tilde{q}}^T D(q) \dot{\tilde{q}}$$

This in the energy.

$$V(\tilde{q}, \dot{\tilde{q}}) > 0 \qquad \text{(positive definite)}$$

$$\dot{V} = \tilde{q}^T K_p \dot{\tilde{q}} + \frac{1}{2} \ddot{\tilde{q}}^T D(q) \dot{\tilde{q}} + \frac{1}{2} \dot{\tilde{q}}^T \left[\dot{\overline{D(q) \dot{\tilde{q}}}} \right]$$

$$= \tilde{q}^T K_p \dot{\tilde{q}} + \frac{1}{2} \ddot{\tilde{q}}^T D(q) \dot{\tilde{q}} + \frac{1}{2} \dot{\tilde{q}}^T \left[\dot{D}(q) \dot{\tilde{q}} + D(q) \ddot{\tilde{q}} \right]$$

$\neq$ (q^d = constant)

$$\dot{\tilde{q}} = \dot{q}^d - \dot{q} = -\dot{q} \quad , \quad \ddot{\tilde{q}} = -\ddot{q}$$

$$\dot{V} = \tilde{q}^T K_p \dot{\tilde{q}} + \frac{1}{2} \dot{\tilde{q}}^T D(q) \ddot{\tilde{q}} + \frac{1}{2} \dot{\tilde{q}}^T D(q) \ddot{\tilde{q}} + \frac{1}{2} \dot{\tilde{q}}^T \dot{D}(q) \dot{\tilde{q}}$$

$$\dot V = \tilde q^T K_p \dot{\tilde q} - \dot{\tilde q}^T D(q)\ddot q + \frac{1}{2}\dot{\tilde q}^T D(q)\dot{\tilde q}$$

$$D(q)\ddot q = -C(q,\dot q)\dot q - G(q) + \tau$$

then

$$\dot V = \tilde q^T K_p \dot{\tilde q} - \dot{\tilde q}^T\left[-C(q,\dot q)\dot q - G(q) + \tau\right] + \frac{1}{2}\dot{\tilde q}^T \dot D(q)\dot{\tilde q}$$

$$\dot V = -\tilde q^T K_p \dot q + \dot q^T\left[-C(q,\dot q)\dot q - G(q) + \tau\right] + \frac{1}{2}\dot q^T \dot D(q)\dot q$$

$$\dot V = -\tilde q^T K_p \dot q + \dot q^T\left[\frac{1}{2}D(q) - C(q,\dot q)\right]\dot q + \dot q^T\left[-G(q) + \tau\right]$$

$$\dot V = -\tilde q^T K_p \dot q + \frac{1}{2}\dot q^T\left[\dot D(q) - 2C(q,\dot q)\right] + \dot q^T\left[\tau - G(q)\right]$$

$= 0$

$$\dot V = -\tilde q^T K_p \dot q + \dot q^T\left[\tau - G(q)\right]$$

$$\tau = K_p \tilde q - K_D \dot q + G(q)$$

$$\dot V = -\dot q^T K_p \dot q \le 0$$

$K_D = K_D^T > 0$
$K_p = K_p^T > 0$) Positive definite matrices

$\dot{V} \to 0$

$\dot{q} \to 0$

$\ddot{q} \to 0$

$$\tau = G(q)$$

$$\tau = K_p \tilde{q} + G(q)$$

$$\tilde{q} = 0$$

For industrial design we say $G(q) = 0$

$$\tau = K_p \tilde{q} - K_D \dot{q}$$

$$\tilde{q} \to \epsilon$$

ϵ becomes small i

$$\tau = K_p \tilde{q} - \frac{K_D}{1 - \tau s} [q]$$